Is your robot motion jerky?

Generalizing Trajectory Retiming to Quadratic Objective Functions

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Introduction

Trajectory Retiming:
compute a feasible speed profile to execute a path

Example Applications:
Decoupled approach to Motion Planning

Path Planning	→	Trajectory Retiming
Compute a collision-free, kinematically feasible path		*Compute dynamically feasible timestamps*

Predefined path (e.g. painting, machining)

Problem Formulation

Path: $\boldsymbol{q}(s): [0,1] \to \mathbb{R}^n$
Parameterization: $s(t): [0,T] \to [0,1]$

$$s^*(t) = \underset{s(t)}{\arg\min} \quad C(s) \quad \longleftarrow \text{objectives}$$

subject to

$$\boldsymbol{A}(s)\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{B}(s)\dot{\boldsymbol{q}} + \boldsymbol{f}(s) \in \mathscr{C}(s),$$
$$\boldsymbol{A}^v(s)\dot{\boldsymbol{q}} + \boldsymbol{f}^v(s) \in \mathscr{C}^v(s)$$

Dynamics & state/control limits

Related Works

Time-Optimal Path Parameterization (TOPP)
Minimize trajectory duration ($C(s) \coloneqq T$)

The Problem
Bang-bang solution saturates control limits
- ➛ No margin for closed-loop controller
- ➛ Cannot handle secondary objectives

Approach

Instead of maximizing speed, let's minimize quadratic objectives!

$$s^*(t) = \underset{s(t)}{\arg\min} \quad \|SpeedObjective\|^2 + \|ControlEffort\|^2 + \ldots$$
$$\text{subject to} \quad Dynamics$$
$$Control/State\ Limits$$

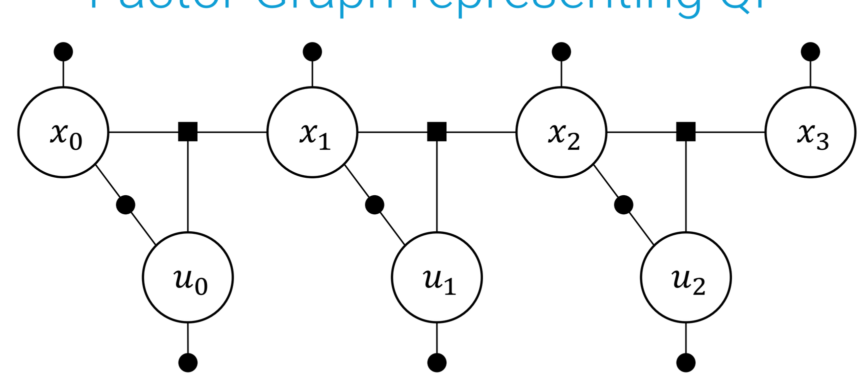
With a general quadratic objective function, we can balance multiple objectives such as max speed (min time), match target speed, max control margin, min control effort, etc.

We call this "QOPP": Quadratic Objective Path Parameterization

Sparse QP

$$\boldsymbol{x}^*, \boldsymbol{u}^* = \underset{u_0,\ldots,x_N}{\arg\min} \sum_{k=0}^{N} Q_k x_k^2 + R_k x_k u_k + N_k u_k^2$$
subject to
$$\boldsymbol{a}_k u_k + \boldsymbol{b}_k x_k + \boldsymbol{c}_k \in \mathscr{C}_k, \quad k = 0,\ldots,N,$$
$$x_{k+1} - x_k - 2u_k\Delta_s = 0, \quad k = 0,\ldots,N-1,$$
$$x_k > 0, \quad k = 0,\ldots,N$$

Factor Graph representing QP



Animated elimination procedure

How to solve QOPP

1. Transcribe into sparse QP using standard TOPP parameterization & discretization
2. Solve sparse QP using factor graph variable elimination[2]

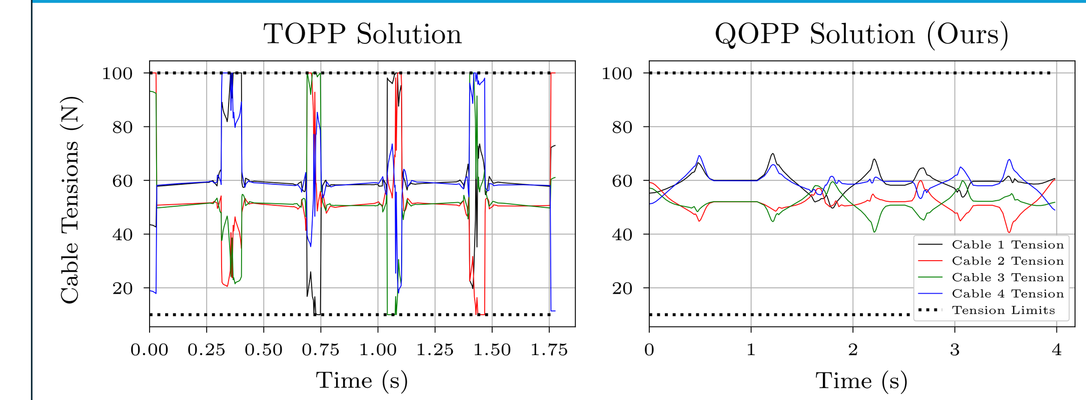
The factor graph depicts the sparsity pattern of the problem.

Performing elimination: During elimination, we only ever need to do 2 types of operations:
- Eliminate $\boldsymbol{u}_k$: Re-write equality constraint & substitute
- Eliminate $\boldsymbol{x}_k$: Solve 2-var parametric piecewise QP

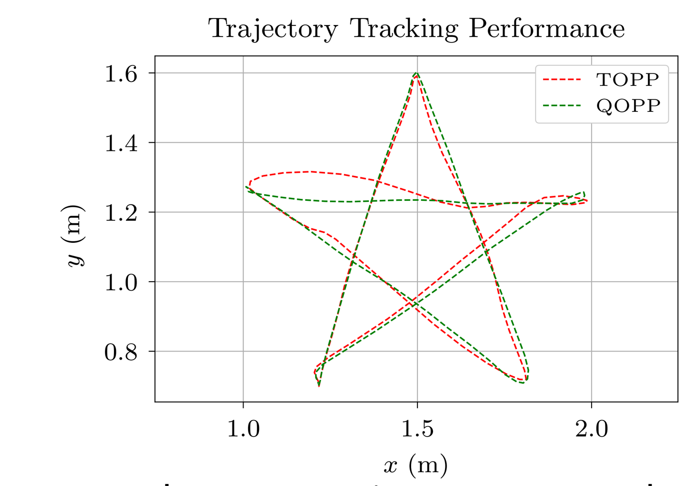
Thanks to the special structure we leverage with factor graphs,

we can solve QOPP in $\mathcal{O}(n)$ time!

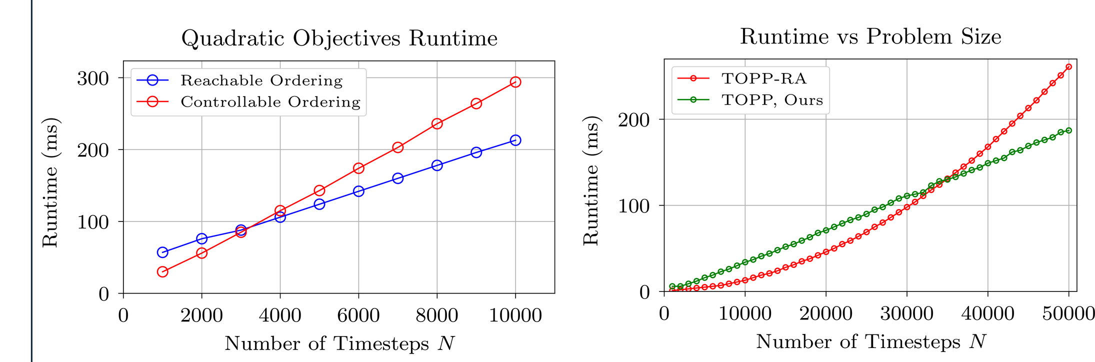
Results



TOPP constantly hits control limits. Meanwhile, QOPP allows us to trade-off speed for control margin.



As a result, executing on a real robot, TOPP has poor tracking performance (QOPP is still good).



QOPP is as-fast or faster than TOPP (C++)
Runtime is $\mathcal{O}(n)$ w.r.t. trajectory length

Conclusions

By balancing multiple objectives & constraints, in practice QOPP achieves better tracking performance and can be solved as-fast or faster than TOPP.

Select References

[TOPP-RA]: H. Pham and Q.-C. Pham, "A New Approach to Time-Optimal Path Parameterization Based on Reachability Analysis," TRO, 2018.
[2]: S. Yang, G. Chen, Y. Zhang, H. Choset, and F. Dellaert, "Equality Constrained Linear Optimal Control With Factor Graphs," ICRA, 2021.