Differential Equation Solution Methods

1. Find a general solution to the following differential equations.

   (a) $4x^3 + 2xy^2 + \left(2x^2 y + 4y^3\right)\dfrac{dy}{dx} = 0$

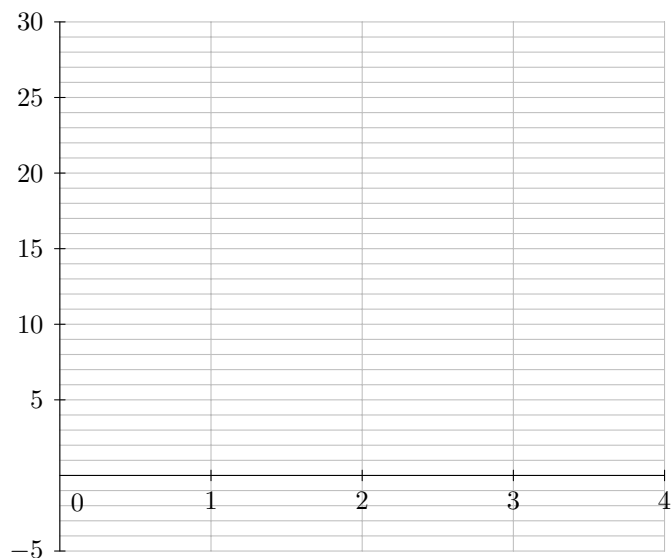   (b) $y^3 \dfrac{dy}{dx} = \left(y^4 + 1\right)\cos x$

(c) $\left(\dfrac{y}{x} + 6x\right) + \dfrac{dy}{dx}(\ln x - 2) = 0$

(d) $y' + 5y = e^{-x}\sin x$

2. Estimate y(3) using Euler's method with a stepsize of $\Delta t = 1$ for the differential equation

$$\frac{dy}{dt} = 3t - y, \quad y(0) = -2.$$

(a) Plot your solution on the graph below.

(b) Find the solution $y(t)$ to the initial value problem and plot it on the graph as well. Does Euler's method give and overestimate or underestimate?

3. Find the solutions to the following initial value problems:

(a) $y'' + y' = 1 + 2e^{-x}$, $y(0) = y'(0) = 1$

(b) $y'' + 2y' + 5y = 0$, $y(0) = 1, y'(0) = 2$

Undetermined Coefficients Review

4. Find the form of the particular solution for the following differential equations.

   (a) $y'' + 4y' + 4y = \sin x + x^2 e^{-2x}$

   (b) $y''' + 4y'' + 5y' = x^3 + e^{2x} \sin x$

   (c) $y'''' + 2y'' + y = \sin x + x^2 \cos x$

Variation of Parameters

5. Solve the following differential equations.

   (a) $y'' - 4y = \sinh(2x)$

   (b) $y'' + 4y = \sin^2 x$