Heat Equation

1. Solve the following heat equation.

$$u_t = u_{xx}, \quad 0 < x < 10, \quad 0 < t$$
$$u(0,t) = 20, \quad u(10,t) = 40$$
$$u(x,0) = 60, \quad 0 < x < 10$$

Wave Equation

2. Solve the wave equation with Dirichlet boundary conditions

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < L, \quad t > 0$$
$$u(0,t) = u(L,t) = 0, \quad t > 0$$
$$u(x,0) = L(1-x), \quad u_t(x,0) = 0, \quad 0 < x < L$$

(a) We can either look for all solutions of the form $u(x,t) = X(x)T(t)$, or recognize the boundary conditions as Dirichlet boundary conditions, and look for solutions of the form
$$u(x,t) = \sum_n c_n(t)\phi_n(x).$$
What are the functions $\phi_n(x)$? What eigenvalue problem do they solve?

(b) What differential equation must $c_n(t)$ solve?

(c) Find $c_n(t)$.

(d) How do we match up the initial conditions $u(x,0) = L(1-x)$ and $u_t(x,0) = 0$?

1. Solve the following wave equation.

$$u_{tt} = 4u_{xx}, \quad 0 < x < 10, \quad 0 < t$$
$$u_x(0,t) = 0, \quad u_x(10,t) = 0$$
$$u(x,0) = 40 - 2x, \quad 0 < x < 10$$
$$u_t(x,0) = 2x - 40, \quad 0 < x < 10$$