

Power Series

2. Show Euler's formula,

$$e^{i\theta} = \cos \theta + i \sin \theta,$$

by using the Taylor series for e^x , $\cos x$, and $\sin x$.

3. The power series

$$1 - x + x^2 - x^3 + \cdots = \frac{1}{1 - (-x)}$$

can be thought of as a geometric series with multiplier $-x$.

(a) For what values of the multiplier x does the series converge?

(b) The derivative of $\ln(1 + x)$ is $\frac{1}{1 + x}$. Use the series above to derive a power series for $\ln(1 + x)$ by integrating the series term by term.

4. Determine a power series solution to the following linear initial value problem.

$$y' = (x - 1)^2 y, \quad y(1) = -1$$