

## Laplace Transform

1. Solve the following initial value problems using the Laplace Transform

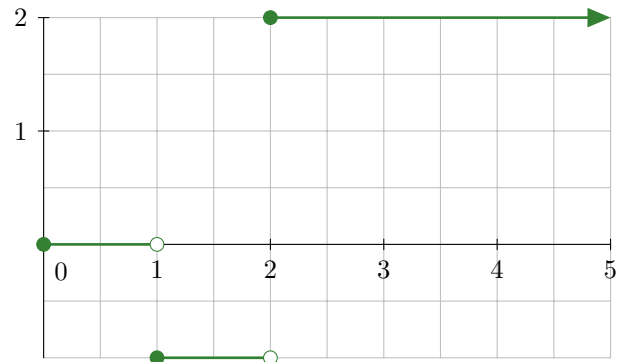
(a)  $y'' + 3y' + 2y = \sin x, \quad y(0) = 1, y'(0) = 2$

(b)  $y'' + 3y' + 2y = x^2e^{-x}, \quad y(0) = 3, y'(0) = -1$

2. Find  $\mathcal{L}^{-1}\left\{\frac{1}{(x-4)(x+1)}\right\}$  by taking a convolution.

3. Solve the following initial value problem (piece-wise function shown below).

$$y'' + 2y' + y = \begin{cases} 0 & 0 \leq t < 1 \\ -1 & 1 \leq t < 2 \\ 2 & 2 \leq t \end{cases}, \quad y(0) = 1, y'(0) = -1.$$



4. Solve the following initial value problem

$$y'' - 3y' - 4y = \delta(t - 1) - 2\delta(t - 2), \quad y(0) = 2, y'(0) = -1$$

## Laplace Transform Table

| $f(t) = \mathcal{L}^{-1}\{F(s)\}$         | $F(s) = \mathcal{L}\{f(t)\}$                        |
|---|---|
| 1   | $\frac{1}{s}, \quad s > 0$                          |
| $e^{at}$                                  | $\frac{1}{s-a}, \quad s > a$                        |
| $t^n, n = \text{positive integer}$        | $\frac{n!}{s^{n+1}}, \quad s > 0$                   |
| $t^p, \quad p > -1$                       | $\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$          |
| $\sin at$                                 | $\frac{a}{s^2 + a^2}, \quad s > 0$                  |
| $\cos at$                                 | $\frac{s}{s^2 + a^2}, \quad s > 0$                  |
| $\sinh at$                                | $\frac{a}{s^2 - a^2}, \quad s >  a $                |
| $\cosh at$                                | $\frac{s}{s^2 - a^2}, \quad s >  a $                |
| $e^{at} \sin bt$                          | $\frac{b}{(s-a)^2 + b^2}, \quad s > a$              |
| $e^{at} \cos bt$                          | $\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$            |
| $t^n e^{at}, n = \text{positive integer}$ | $\frac{n!}{(s-a)^{n+1}}, \quad s > a$               |
| $u_c(t)$                                  | $\frac{e^{-cs}}{s}, \quad s > 0$                    |
| $u_c(t)f(t-c)$                            | $e^{-cs}F(s)$                                       |
| $e^{ct}f(t)$                              | $F(s-c)$  |
| $f(ct)$                                   | $\frac{1}{s}F\left(\frac{s}{c}\right), \quad c > 0$ |
| $\int_0^t f(t-\tau)g(\tau) d\tau$         | $F(s)G(s)$  |
| $\delta(t-c)$                             | $e^{-cs}$   |
| $f^{(n)}(t)$                              | $s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$     |
| $(-t)^n f(t)$                             | $F^{(n)}(s)$  |