Dot Product with a unit vector

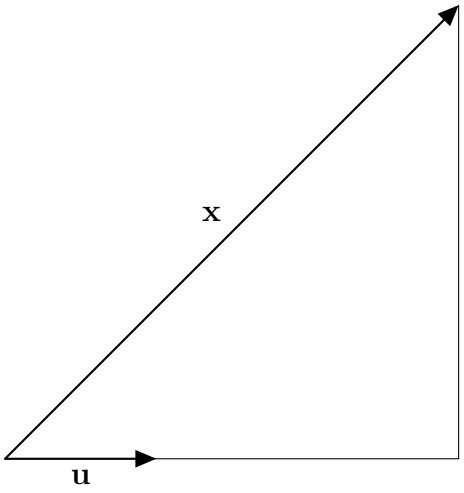
1. The dot product of two vectors $\mathbf{x}$ and $\mathbf{y}$ can be defined as

$$\mathbf{x} \cdot \mathbf{y} = |\mathbf{x}||\mathbf{y}| \cos\theta,$$

where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$.

(a) Show that if $\mathbf{y} = \mathbf{u}$ is a unit vector, $\mathbf{x} \cdot \mathbf{u} = |\mathbf{x}| \cos\theta$.

(b) Draw this length on the figure below and describe the length.



2. Use the idea from the previous page to find the shortest distance from the plane $x + 2y - 2z = 6$ to the point (-2, -2, -1).

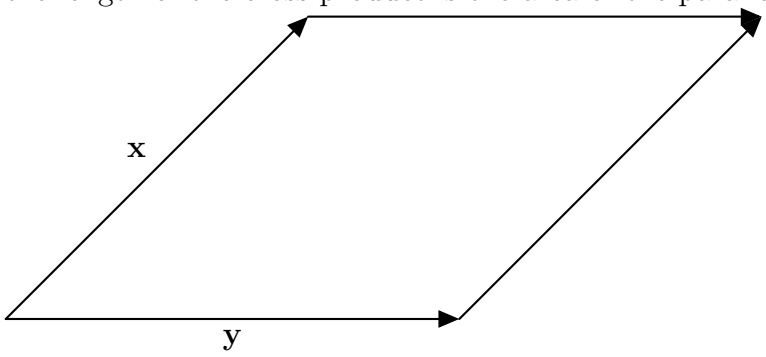
Cross Product

3. The cross product, unlike the dot product, is a vector, which has a length and a direction. The length of the cross product of two vectors $\mathbf{x}$ and $\mathbf{y}$ can be defined as

$$|\mathbf{x} \times \mathbf{y}| = |\mathbf{x}||\mathbf{y}| \sin\theta,$$

where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$. The direction of the cross product is perpendicular to both $\mathbf{x}$ and $\mathbf{y}$.

(a) Given the vectors in the figure below, label the length $|\mathbf{x}| \sin\theta$. Explain why the length of the cross product is the area of the parallelogram.



(b) Does the direction of $\mathbf{x} \times \mathbf{y}$ point into the page or out of the page? What about $\mathbf{y} \times \mathbf{x}$?

4. Use your understanding of the dot product and cross product to solve the following problems.

   (a) Find an equation for the line of intersection of the planes $x + 2y - 2z = 6$ and $2x - y + 2z = 12$.

   (b) Find the distance between the skew lines parameterized by $\mathbf{r}(t) = \langle 1 + 2t, 2 - 2t, 3 + t \rangle$ and $\mathbf{s}(t) = \langle -1 - 2t, -2 + 2t, -3 - t \rangle$.

Level Sets and Graphs

5. Given the plane $x + 2y + 3z = 0$,

   (a) Find a parameterization $\mathbf{r}(x, y) : \mathbb{R}^2 \to \mathbb{R}^3$ for this plane.

6. Write down a function $g : \mathbb{R}^3 \to \mathbb{R}^1$.

   (a) The graph of g is a subset of _____.

   (b) A level set of g is a subset of _____.

7. Find the level surface of the function $f(x, y, z) = x^2 + y^2 + z^2$ that goes through the point $(1, 2, 2)$. Draw a sketch of this surface.

8. Given the function $g(x, y, z) = x^2 + y^2 - z^2$.

   (a) Find the level surface that goes through the point $(3, 4, 5)$.

   (b) Find the level surface that goes through the point $(5, 0, 0)$.

   (c) Find the level surface that goes through the point $(0, 0, 4)$.

   (d) Draw a sketch of all three of these surfaces on the same axes.