Dot Product with a unit vector

1. The dot product of two vectors $\mathbf{x}$ and $\mathbf{y}$ can be defined as

$$\mathbf{x} \cdot \mathbf{y} = |\mathbf{x}||\mathbf{y}| \cos \theta,$$

where $\theta$ is the angle between $\mathbf{x}$ and $\mathbf{y}$.

(a) Show that if $\mathbf{y} = \mathbf{u}$ is a unit vector, $\mathbf{x} \cdot \mathbf{u} = |\mathbf{x}| \cos \theta$.

(b) Draw this length on the figure below and describe the length.
2. Use the idea from the previous page to find the shortest distance from the plane \( x + 2y - 2z = 6 \) to the point \((-2, -2, -1)\).
Cross Product

3. The cross product, unlike the dot product, is a vector, which has a length and a direction. The length of the cross product of two vectors \( \mathbf{x} \) and \( \mathbf{y} \) can be defined as

\[
|\mathbf{x} \times \mathbf{y}| = |\mathbf{x}| |\mathbf{y}| \sin \theta,
\]

where \( \theta \) is the angle between \( \mathbf{x} \) and \( \mathbf{y} \). The direction of the cross product is perpendicular to both \( \mathbf{x} \) and \( \mathbf{y} \).

(a) Given the vectors in the figure below, label the length \( |\mathbf{x}| \sin \theta \). Explain why the length of the cross product is the area of the parallelogram.

(b) Does the direction of \( \mathbf{x} \times \mathbf{y} \) point into the page or out of the page? What about \( \mathbf{y} \times \mathbf{x} \)?
4. Use your understanding of the dot product and cross product to solve the following problems.

(a) Find an equation for the line of intersection of the planes $x + 2y - 2z = 6$ and $2x - y + 2z = 12$.

(b) Find the distance between the skew lines parameterized by $\mathbf{r}(t) = \langle 1 + 2t, 2 - 2t, 3 + t \rangle$ and $\mathbf{s}(t) = \langle -1 - 2t, -2 + 2t, -3 - t \rangle$. 
Level Sets and Graphs

5. Given the plane \( x + 2y + 3z = 0 \),

(a) Find a parameterization \( r(x, y) : \mathbb{R}^2 \to \mathbb{R}^3 \) for this plane.

6. Write down a function \( g : \mathbb{R}^3 \to \mathbb{R}^1 \).

(a) The graph of \( g \) is a subset of ________.

(b) A level set of \( g \) is a subset of ________.

7. Find the level surface of the function \( f(x, y, z) = x^2 + y^2 + z^2 \) that goes through the point \((1, 2, 2)\). Draw a sketch of this surface.
8. Given the function $g(x, y, z) = x^2 + y^2 - z^2$.

(a) Find the level surface that goes through the point $(3, 4, 5)$.

(b) Find the level surface that goes through the point $(5, 0, 0)$.

(c) Find the level surface that goes through the point $(0, 0, 4)$.

(d) Draw a sketch of all three of these surfaces on the same axes.