Parameterization, Curvature

1. Given the parameterization

\[ \mathbf{r}(t) = (2 \cos(t), 3 \sin(t)), \quad 0 \leq t \leq 2\pi. \]

(a) Show that \( x(t) \) and \( y(t) \) satisfy the equation for the ellipse

\[ \frac{x^2}{4} + \frac{y^2}{9} = 1. \]

(b) Draw this ellipse on the axes below.

(c) Find \( \mathbf{v} = \mathbf{r}'(0) \) and \( \mathbf{a} = \mathbf{r}''(0) \) and find \( \kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} \) at \( t = 0 \).
2. Given the parameterization for a spiral,

\[ \mathbf{r}(t) = (\cos t, \sin t, t). \]

(a) Find the unit tangent vector \( \mathbf{T} \) and the unit normal vector \( \mathbf{N} \) at time \( t = 1 \).

[Hint: you can find the normal direction by taking \( \mathbf{v} \times \mathbf{a} \times \mathbf{v} \).]

(b) Find an equation for a line tangent to the spiral at time \( t_0 \).

(c) Given \( t_0 = \pi \), find the point at which the tangent line intersects the plane \( z = 0 \).
3. Give examples of the following.

(a) An equation for a cylinder so that the point \((1, 2, 3)\) is on its surface.

(b) The point \(P\) on the plane \(3x + 2y + z = 6\) that is closest to the point \(Q(7, 7, -1)\).

(c) A paraboloid that opens downwards in the \(z\) direction and intersects the \(xy\)-plane in the ellipse \(4x^2 + 9y^2 = 36\).

(d) Two parallel lines \(L_1\) and \(L_2\) in the parallel planes \(x + 2y + 2z = 3\) and \(x + 2y + 2z = 6\) so that the distance between \(L_1\) and \(L_2\) is 3.
4. Given the points $A(1, 2, 3)$ and $B(5, 4, -2)$.

(a) Find both the parametric and symmetric equations of the straight line connecting $A$ and $B$.

(b) Find the plane perpendicular to this line that goes through the point $(1, 2, 4)$.

(c) Find a plane that this line does not intersect.