

Parameterization, Curvature

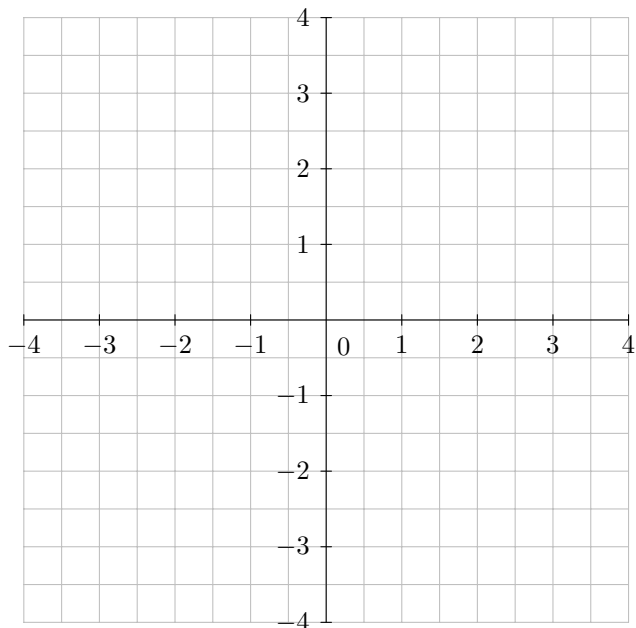
1. Given the parameterization

$$\mathbf{r}(t) = \langle 2 \cos(t), 3 \sin(t) \rangle, \quad 0 \leq t \leq 2\pi.$$

- (a) Show that $x(t)$ and $y(t)$ satisfy the equation for the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

- (b) Draw this ellipse on the axes below.



- (c) Find $\mathbf{v} = \mathbf{r}'(0)$ and $\mathbf{a} = \mathbf{r}''(0)$ and find $\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$ at $t = 0.$,

2. Given the parameterization for a spiral,

$$\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle.$$

(a) Find the unit tangent vector \mathbf{T} and the unit normal vector \mathbf{N} at time $t = 1$.

[Hint: you can find the normal direction by taking $\mathbf{v} \times \mathbf{a} \times \mathbf{v}$.]

(b) Find an equation for a line tangent to the spiral at time t_0 .

(c) Given $t_0 = \pi$, find the point at which the tangent line intersects the plane $z = 0$.

3. Give examples of the following.

(a) An equation for a cylinder so that the point $(1, 2, 3)$ is on its surface.

(b) The point P on the plane $3x + 2y + z = 6$ that is closest to the point $Q(7, 7, -1)$.

(c) A paraboloid that opens downwards in the z direction and intersects the xy -plane in the ellipse $4x^2 + 9y^2 = 36$.

(d) Two parallel lines L_1 and L_2 in the parallel planes $x + 2y + 2z = 3$ and $x + 2y + 2z = 6$ so that the distance between L_1 and L_2 is 3.

4. Given the points $A(1, 2, 3)$ and $B(5, 4, -2)$.
- (a) Find both the parametric and symmetric equations of the straight line connecting A and B .
- (b) Find the plane perpendicular to this line that goes through the point $(1, 2, 4)$.
- (c) Find a plane that this line does not intersect.