Limits

1. Evaluate \( \lim_{(x,y) \to (0,0)} \frac{x^2 y^3}{x^5 + y^5} \).

2. Evaluate \( \lim_{(x,y) \to (0,0)} \frac{x^2 y^3}{x^2 + y^2} \).
Vector Functions

3. For each of the following functions, first find the dimensions of the domain and the range. [For example, \( \mathbf{r}(t) = (t, t^2, t^3) : \mathbb{R} \to \mathbb{R}^3 \).] Then find the partial derivatives of each component with respect to \( t \), \( u \), and/or \( v \).

   (a) \( \mathbf{r}(u,v) = (x(u,v), y(u,v), z(u,v)) = (u, v, 9 - u^2 - v^2) \)

   (b) \( \mathbf{r}(t,u,v) = (x(t,u,v), y(t,u,v)) = (\sin(t - 2v), \sqrt{u + 3v}) \)

4. Given the plane \( x + 2y + 3z = 0 \), find a parameterization \( \mathbf{r}(x,y) : \mathbb{R}^2 \to \mathbb{R}^3 \) for this plane.
The Chain Rule

5. Related Rates: Gasoline is pouring into a tank in the shape of a cone of radius 3 feet and height 4 feet. When the depth of the gasoline is 2 feet, the depth is increasing at 0.2 ft/sec. How fast is the volume of the gasoline changing at that instant?
6. Let \( z = f(x, y) \) and let \( x = r \cos \theta \) and \( y = r \sin \theta \).

(a) Find \( \frac{\partial z}{\partial r} \).

(b) Find \( \frac{\partial^2 z}{\partial r^2} \).

7. Find a function \( z = f(x, y) \) with \( x = r \cos \theta \) and \( y = r \sin \theta \) so that
\[
\frac{\partial^2 z}{\partial r \partial \theta} = -2x r \sin \theta \cos \theta - x^2 \sin \theta + 2y r \cos \theta \sin \theta + y^2 \cos \theta.
\]