

2. Give examples of the following, or explain why no such example exists.
- (a) An equation for a cylinder so that the point $(1, 2, 3)$ is on its surface.
- (b) A paraboloid that opens downwards in the z direction and intersects the xy -plane in the ellipse $4x^2 + 9y^2 = 36$.
- (c) Two parallel lines L_1 and L_2 in the parallel planes $x + 2y + 2z = 3$ and $x + 2y + 2z = 6$ so that the distance between L_1 and L_2 is 3.
- (d) Two skew lines L_1 and L_2 that sit in parallel planes $2x + 2y + z = 5$ and $2x + 2y + z = -4$ so that the minimum distance between L_1 and L_2 is 9.

3. Given the parameterization $\mathbf{r}(t) = \langle \cos t, \sin t, 3t \rangle$.

(a) Find the unit tangent vector at time t .

(b) Find an equation for the line tangent to the curve at $t = \pi$.

(c) Calculate the curvature κ of the curve at $t = \pi$.

(d) Where does this line intersect the plane $z = 0$.

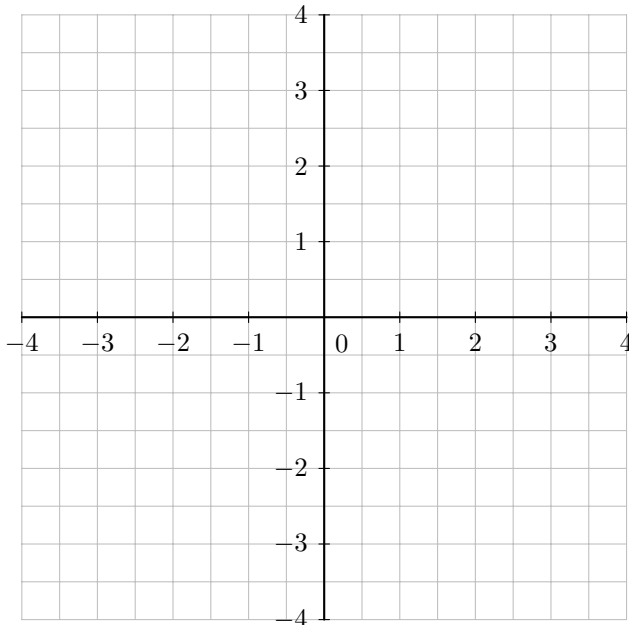
4. For the following parameterization $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$
- (a) Find the velocity and acceleration vectors at time $t = 1$, $\mathbf{v}(1) = \mathbf{r}'(1)$ and $\mathbf{a}(1) = \mathbf{r}''(1)$.
- (b) Find the unit tangent vector \mathbf{T} and the unit normal vector \mathbf{N} at time $t = 1$.
[Hint: you can find the normal direction by taking $\mathbf{v} \times \mathbf{a} \times \mathbf{v}$.]
- (c) Find the curvature κ at time $t = 1$.

The Gradient

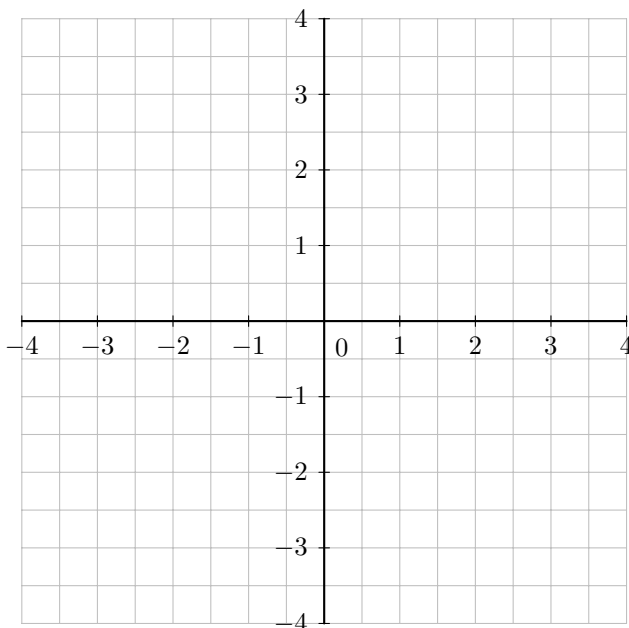
5. Given each of the following equations,

- (a) On the axes on the right, draw level curves of the function $f(x, y)$.
- (b) Find the gradient of f , $\langle f_x, f_y \rangle$, and draw some gradient vectors on the graph of the level curves. What is true about the gradient and level curves?

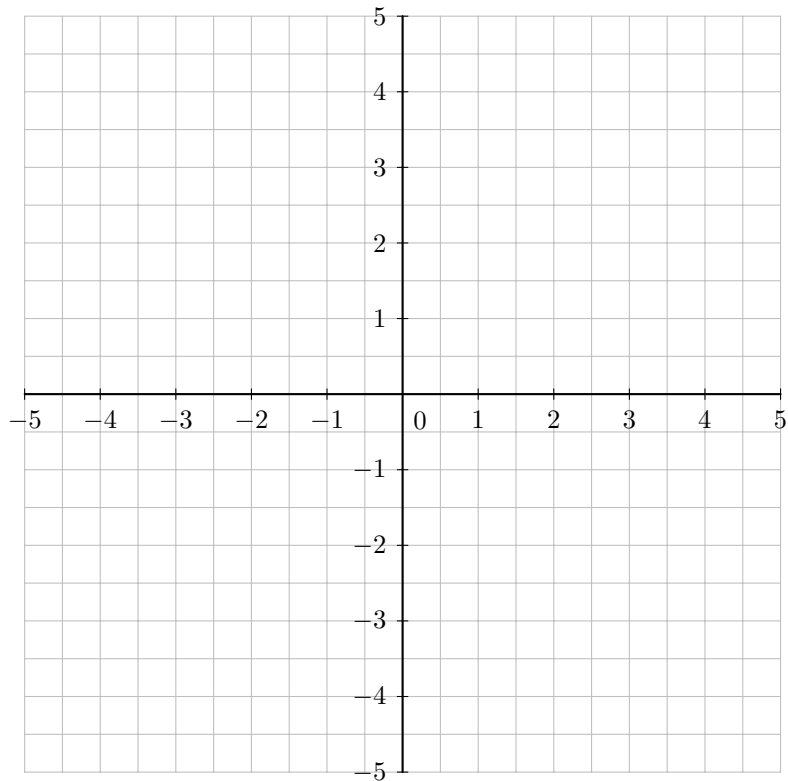
- $f(x, y) = x + y + 3$



- $f(x, y) = x^2 + y^2$



- (c) Show on the graph below that the points where the level curves of $f(x, y) = x + y + 3$ are tangent to the curve $x^2 + y^2 = 8$ are $(2, 2)$ and $(-2, -2)$.



- (d) Show that the gradient of $f(x, y) = x + y + 3$ and the gradient of $g(x, y) = x^2 + y^2$ are parallel (or antiparallel) at $(2, 2)$ and $(-2, -2)$.

- (e) Are these two points the maximum and minimum value of $f(x, y) = x + y + 3$ under the constraint $g(x, y) = x^2 + y^2 = 8$?