The Chain Rule

1. Related Rates: Gasoline is pouring into a tank in the shape of a cone (inverted cone? point at the bottom) of radius 3 feet and height 4 feet. When the depth of the gasoline is 2 feet, the depth is increasing at 0.2 ft/sec. How fast is the volume of the gasoline changing at that instant?

(a) First write out the equation of the volume of a cone. Which variables are changing with time?

(b) Differentiate implicitly with respect to time.

(c) Solve for the rate of change of the volume.
2. If \( z = x^2 y^2 \) and \( x = u - v \) and \( y = u + v \).

(a) Find \( \frac{\partial x}{\partial u} \) and \( \frac{\partial y}{\partial u} \).

(b) Write down \( \frac{\partial z}{\partial u} \) in terms of \( \frac{\partial x}{\partial u} \) and \( \frac{\partial y}{\partial u} \).

(c) Solve for \( \frac{\partial z}{\partial u} \) in terms of \( u \) and \( v \).

(d) Find \( \frac{\partial^2 z}{\partial u \partial v} \).
3. Let \( z = f(x, y) \) and let \( x = r \cos \theta \) and \( y = r \sin \theta \).

   (a) Find \( \frac{\partial z}{\partial r} \).

   (b) Find \( \frac{\partial^2 z}{\partial r^2} \).

4. Find a function \( z = f(x, y) \) with \( x = r \cos \theta \) and \( y = r \sin \theta \) so that

   \[
   \frac{\partial^2 z}{\partial r \partial \theta} = -2xr \sin \theta \cos \theta - x^2 \sin \theta + 2yr \cos \theta \sin \theta + y^2 \cos \theta.
   \]
5. Given the function

\[ g(t, x, y) = y \cos (xt), \]

with \( y(x) = e^x \) and \( x(t) = \sin t \).

(a) Find \( \frac{\partial g}{\partial y} \).

(b) Find \( \frac{\partial g}{\partial x} \).

(c) Find \( \frac{\partial g}{\partial t} \).