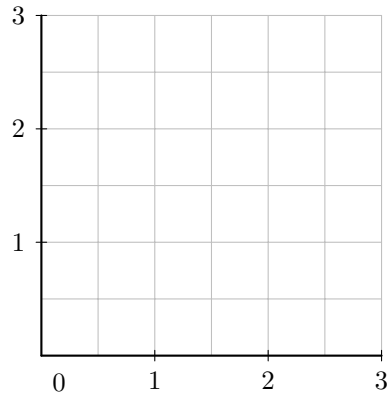
# Optimization

1. Given the function,

$$f(x, y) = 2xy - x^2y - y^2x.$$

(a) Find all the critical points. The graph below might be helpful.

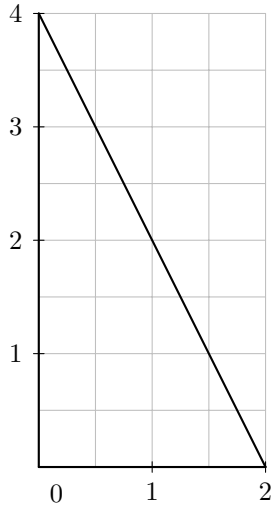


$$f(x, y) = 2xy - x^2y - y^2x$$

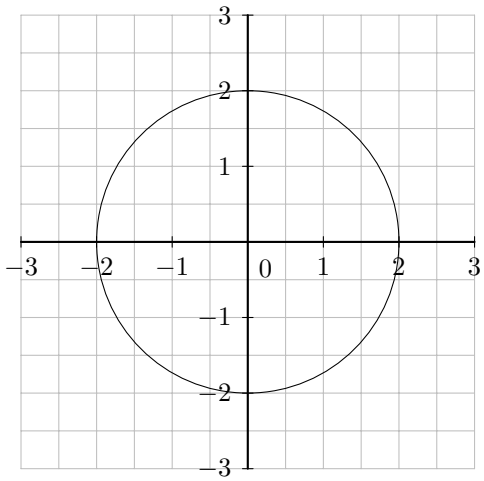
(b) Classify each critical point using the Hessian matrix.

$$f(x, y) = 2xy - x^2y - y^2x$$

(c) Find the maximum and minimum values of the function in the region within the triangle shown below, including the edges of the triangle.



2. Given the function $f(x, y) = 5x^2 - 6xy + 5y^2$ with constraint $x^2 + y^2 = 4$, shown below.




(a) Find all points where the gradient of $f$ is parallel (or anti-parallel ) to the gradient of $g(x, y) = x^2 + y^2$.

(b) Find all points on the constraint $x^2 + y^2 = 4$ where the gradient of $f$ is parallel (or anti-parallel ) to the gradient of $g(x, y) = x^2 + y^2$.

(c) Find the maximum and minimum values of $f(x, y)$ under the constraint $x^2 + y^2 = 4$.